PHYSICAL REVIEW E VOLUME 52, NUMBER 6 DECEMBER 1995

Growth of interfaces with strong quenched disorder: Columnar media

J.M. López[1,2] and M.A. Rodríguez[1]
[1] Instituto de Física de Cantabria, Consejo Superior de Investigaciones Científicas—Universidad de Cantabria, E-39005 Santander, Spain
[2] Departamento de Física Moderna, Universidad de Cantabria, E-39005 Santander, Spain
(Received 28 June 1995)

The growth of interfaces through columnar media is analyzed using a simple two-dimensional model in which the pinning force and the diffusivity are modeled by random quenched fields. The characteristic roughness exponents χ and β are analytically obtained in agreement with simulations. It is shown how disorder in the diffusivity, controlled by a parameter $\alpha < 1$, strongly affects the scaling of the interface. Disorder-dependent exponents $\beta = (3-\alpha)/[2(2-\alpha)]$ and $\chi = (3-\alpha)/[2(1-\alpha)]$ are exactly calculated.

PACS number(s): 47.55.Mh, 05.40.+j, 05.60.+w, 64.60.Fr

I. INTRODUCTION

The growth of interfaces through random media with quenched disorder is a subject of recent interest that has attracted the attention of much research. The reason is twofold: on the one hand, there is a considerable dispersion of results obtained from experiments, simulations, and theory [1] and, on the other hand, the problem is relevant in fields with important technical applications [2,3] such as surface growth and deposition processes, wetting in porous media, directed polymers in random media, pinning of the vortex lines in type-II superconductors, and charge density waves.

The dynamics of interfaces with thermal fluctuations has been described by means of stochastic differential equations of Kardar-Parisi-Zhang (KPZ) [4] and Edwards-Wilkinson (EW) [5] type with a spatiotemporal noise $\eta(x,t)$. Considerable agreement between theory and simulations of growth models usually has been obtained in the case of dynamical disorder. A number of growth models (ballistic deposition, deposition with diffusion, and the Eden model among others) in which thermal fluctuations are relevant can be described by the KPZ (or EW) equation [6].

The situation in the case of quenched disorder is very different. Quenched disorder usually appears in fluid invasion of porous media [7] or domain-wall motion with random-field (or random-bond) disorder [8]. The interface is driven through a disordered medium by an external driving force F. There exist similar models with stochastic equations, the so-called quenched KPZ and quenched EW equations [8–13], but now the noise is quenched and dependent on the height of the interface $h(x,t)$ as $\eta(x,h)$. The calculation of roughness exponents in this case is a complicated mathematical problem since the nonlinearity is included in the disorder. However, several points concerning the growth of interfaces modeled in this manner have been clarified, mainly by means of simulations [9–11] and phenomenological arguments [8,11,12]. The motion of the interface is dominated by the pinning forces present in the inhomogeneous medium. These pinning forces are able to slow down the motion in large regions. There exists a critical force F_c [8–12] separating two different regimes. Above F_c, for $F \geq F_c$, the quenched disorder is relevant and the exponents corresponding to the critical point can be calculated [8,12]. In the strong pushing limit, where $F \gg F_c$, the interface moves faster and behaves as in the dynamical disordered case $\eta(x,h(x,t)) \sim \eta(x,Ft)$ [11]. On the contrary, for $F < F_c$ the interface remains pinned by the disorder. It has been shown [10,13] that at least three different universality classes can be distinguished in the quenched KPZ equation, although experiments, simulations, and theory near this point $F \simeq F_c$ are not yet in good agreement (see [1] for a recent review).

It is well known that the typical exponents can change in the KPZ equation with a long-range correlated noise and nonuniversal correlation-dependent exponents are found [14]. Also, there is numerical evidence of the great influence on the roughness of the interfaces by some kinds of quenched disorder [1,15,16]. In this case, disorder can affect the concept of universality that one expects to exist. In general, it is a very interesting problem determining how disorder can change the universal behavior.

In this paper we deal with a different type of quenched disorder, quenched columnar disorder, that illustrates some other possibilities of growth and is relevant in some porous media [17]. Incompressible flows in porous materials are usually modeled by classical diffusion of independent particles in random quenched environments. In such a situation, long-range correlations could happen if the flow lines of the invading fluid are correlated over large distances, which occurs in the study the motion of a fluid in a stratified porous media [18]. Then, the quenched disorder is taken constant along vertical channels $\eta(x)$ (i.e., columnar disorder). In the problem of two-phase flow of viscous fluids in porous media, the effect of columnar disorder has been also analyzed [19]. Quenched columnar disorder also plays an important role in the roughening of directed polymers where columnar defects may compete against point defects [20].

The differences between columnar $\eta(x)$ and uniform quenched $\eta(x,y)$ disorder are considerable. As we will show, the growth is very dependent on each particular realization of disorder and as a consequence several scaling laws that hold in the uniform case here are not valid. Despite that the disorder is in this case stronger than in the uniform case, pinned states do not appear. Pinning is an effect due to the nonlinearity of the noise [8,12]. As expected, roughness in columnar disorder is greater than in uniform disorder. As concerns the growth of interfaces we can say that the dynamical disorder $\eta(x,t)$ is weak since roughness is small as a result of temporal averaging of forces, the columnar disorder $\eta(x)$ is strong because forces remain frozen in time, and the uniform quenched disorder $\eta(x,h(x,t))$ is an intermediate case.

II. COLUMNAR QUENCHED MODEL AND ANALYTICAL RESULTS

In this paper we shall consider only a linear growth model since it allows the possibility of analytical results. We introduce a model with columnar quenched disorder both in the force term and in the diffusivity.

Disorder in the diffusion coefficient is taken as in the theory of linear transport through random media. When both disorders are uncorrelated the usual analysis can be exactly carried out. In the strong disorder case, the anomalous behavior found in the transport theory here leads to an anomalous roughness exponent. This roughness increases indefinitely as disorder becomes stronger. Simulations are in agreement with this analysis. Hence, with this simple example, we illustrate a mechanism by which a quenched disorder with a well defined correlation length induces anomalous roughness. Then, instead of having a universal behavior, the exponents are dependent on disorder.

Only two effects are taken into account: the pinning force, which is constant along vertical channels, and the interaction between neighboring channels, which is taken in a diffusive manner. The continuous version of this model can be written in terms of a stochastic differential equation for the height $h(x,t)$ of the interface at position x and time t as

$$\frac{\partial}{\partial t}h(x,t) = \frac{\partial}{\partial x}D(x)\frac{\partial}{\partial x}h(x,t) + F + \eta(x), \quad (1)$$

where $D(x) \geq 0$ is the random diffusion coefficient related to the interchange between vertical channels and the driving force F. $\eta(x)$ represents the random pinning forces in the random medium and is taken as a Gaussian noise with mean $\langle\eta(x)\rangle = 0$ and correlation $\langle\eta(x)\eta(x')\rangle = \theta\delta(x-x')$. $D(x)$ is also a white noise, but due to the constraint $D(x) \geq 0$ it cannot be Gaussian.

Since (1) can be seen as a model of diffusing particles with sources and absorbers we shall use for $D(x)$ the same stochastic description as in the theory of transport through random fields. Hence the discrete version of (1) is a master equation

$$\frac{\partial h_i(t)}{\partial t} = (1 - E_i^-)\frac{D_i}{a^2}(E_i^+ - 1)h_i(t) + \eta_i/\sqrt{a} + F, \quad (2)$$

where $E_i^\pm$ is a shift operator, $E_i^\pm f_i = f_{i\pm 1}$, a is the lattice spacing, η_i is a Gaussian uncorrelated noise $\langle\eta_i\eta_j\rangle = \theta\delta_{i,j}$, and D_i is another uncorrelated noise distributed according to a probability density $P(D)$. Usually, this probability density is modeled by taking $P(D) = N_\alpha D^{-\alpha} f_c(D/D_{\max})$, where N_α is the normalization constant and f_c a cutoff function basically specified by the fixed $D_{\max}$. As it has been shown [21–23], this distribution for disorder covers all situations of physical interest. Briefly, when $P(D)$ becomes zero as $D \to 0$ disorder is weak because the horizontal correlation length is diffusive. On the contrary, if $0 < \alpha < 1$, disorder is strong and an anomalous behavior in the diffusion length appears.

In the following we use a reference frame with uniform velocity F along the vertical coordinate, which is equivalent to considering (1) or (2) with $F = 0$. This equivalence comes from the invariance of a columnar field to translations along the vertical coordinate.

As usual, let us define the height and width of the interface averaged over pieces of length l as

$$\overline{h}_\eta(l,t) = \{h(x,t)\}_l,$$

$$\sigma_\eta^2(l,t) = \{(h(x,t) - \{h(x,t)\}_l)^2\}_l, \quad (3)$$

where $\{A\}_l = (1/l)\int_x^{x+l} A dx'$. Obviously these functions are dependent on the realization of disorder. Disorder-independent quantities are obtained when averaging over realizations of disorder

$$\overline{h}(l,t) = \langle\{h(x,t)\}_l\rangle,$$

$$\sigma^2(l,t) = \langle\{(h(x,t) - \{h(x,t)\}_l)^2\}_l\rangle. \quad (4)$$

The usual scaling laws postulate a saturation time $t_s(l) \sim l^z$, where z is called the dynamical exponent. For times $t < t_s$ the width scales with time as

$$\sigma(l,t) \sim t^\beta$$

and after time $t_s(l)$ the width does not depend on time but on the length l of the interface. This dependence on l of the interfacial width is used to define the roughness exponent χ

$$\sigma(l, t > t_s) \sim l^\chi,$$

where $\chi = z\beta$. In the problem of growing interfaces with time-dependent noise (thermal fluctuations) the scaling with size l in the saturation regime can be either a piece of interface of length l or the overall length L. We shall show that this equivalence is no longer valid in our case.

First, we analyze the case in which the interactions between channels (i.e., the diffusivity) are constants. From (1), in the comoving frame $F = 0$ with $D(x) = D$, the continuous model can be easily solved, obtaining

$$h(x,t) = \int_0^t \int_{-\infty}^{\infty} G(x,t|x',t')\eta(x')dx', \quad (5)$$

where

$$G(x,t|x',t') = \frac{1}{\sqrt{4\pi(t-t')D}} \exp\left(-\frac{(x-x')^2}{4Dt}\right)$$

is the Green function of the problem without sources and the initial condition $h(x,t=0)=0$ has been assumed. Now, from (5), we calculate the width averaged over realizations of disorder

$$\sigma^2(l,t) = \frac{\theta t^{3/2}}{4\pi D}\left(\int_0^{l/\sqrt{t}} \frac{f(x,x)}{(l/\sqrt{t})}dx - \int_0^{l/\sqrt{t}}\int_0^{l/\sqrt{t}} \frac{f(x,y)}{(l/\sqrt{t})^2}dxdy\right),$$

with

$$f(x,y) = \int_{-\infty}^{\infty} dx_1 \int_0^1 \frac{d\tau_1}{\sqrt{1-\tau_1}} \int_0^1 \frac{d\tau_2}{\sqrt{1-\tau_2}} \times \exp\left(-\frac{(x-x_1)^2}{4D(1-\tau_1)} - \frac{(x-x_2)^2}{4D(1-\tau_2)}\right).$$

The height averaged over realizations is also obtained in a straightforward manner, $\overline{h}(l,t)=0$. At short times, when $l/\sqrt{t} \gg 1$, the width scales as $\sigma^2(t) \sim t^{3/2}$, which gives immediately the exponent $\beta = 3/4$. Since the saturation time goes like $t_s \sim l^2$, we obtain $\sigma^2(l) \sim l^3$ in the saturation regime and the roughness exponent is $\chi = 3/2$. The temporal dependence of the horizontal characteristic length is diffusive.

In a columnar medium with different interplay between channels depending on the substrate position, the growth of an interface can be strongly influenced by disorder. With our model defined in (1) this dependence can be studied analytically. We can proceed either from the continuous version (1) or from the discretized one (2). In order to follow an analysis similar to the constant diffusion case, we first consider the continuous model (1), taking the solution of $h(x,t)$ in the same manner as in (1), but now the Green function is disorder dependent. Taking into account that $D(x)$ and $\eta(x)$ are uncorrelated, we obtain from definitions (4) and (5) an expression for the interface width in terms of Laplace transforms:

$$\sigma^2(l,s) = \frac{\theta}{l}\int_0^l dx \int_{-\infty}^{\infty} dx_1 \int_0^s \frac{ds_1}{(s-s_1)s_1} \times \langle G_{s-s_1}(x,x_1)G_{s_1}(x,x_1)\rangle - \frac{\theta}{l^2}\int_0^l dx \int_0^l dy \int_{-\infty}^{\infty} dx_1 \int_0^s \frac{ds_1}{(s-s_1)s_1} \times \langle G_{s-s_1}(x,x_1)G_{s_1}(y,x_1)\rangle, \quad (6)$$

where $G_s(x)$ is the Laplace transform of the disorder-dependent Green function.

A method has been introduced recently in Ref. [24] to calculate averaged products of n Green functions in the context of diffusing particles through disordered media. Following [24], the leading contribution of the averaged product of two Green functions is

$$\langle G_s(x,y)G_{s'}(x',y)\rangle \simeq \frac{1}{[4s\Gamma(s)]^{1/2}} e^{-[s/\Gamma(s)]^{1/2}|x-y|} \times \frac{1}{[4s'\Gamma(s')]^{1/2}} e^{-[s'/\Gamma(s')]^{1/2}|x'-y|}. \quad (7)$$

$\Gamma(s)$ is a renormalized frequency-dependent diffusion coefficient given by the condition

$$\left\langle \frac{D_i - \Gamma(s)}{1 - J_s(i,i)[D_i - \Gamma(s)]} \right\rangle = 0 \quad (8)$$

and the function $J_s(i,j) = (E_i^+ - 1)(1 - E_j^+)G_s(i,j)$.

This condition is obtained in a self-consistent manner by assuming an effective medium with memory [24] in the discret version, Eq. (2), of the model. In the process of resummation that leads to Eqs. (7) and (8) it is important to consider the order of limits $a \to 0$ (a being the lattice spacing) and $t \to \infty$. We assume a physical system with small but finite a and consequently we first take the long time limit $t \to \infty$ and then the continuous limit $a \to 0$. In these conditions (8) leads to a frequency-dependent diffusion coefficient $\Gamma(s)$, which scales as $\Gamma(\lambda s) \sim \lambda^\mu \Gamma(s)$, μ being characterized by the disorder [23,24]. Using for the disorder the distribution $P(D) \sim D^{-\alpha}$, from (8) and following [24], we obtain that, for $\alpha < 0$, $\Gamma(0)$ is finite and only a renormalization of the diffusion coefficient $D \approx \Gamma(0)$ occurs, so the horizontal correlation length $l_c(t)$ has a diffusive behavior $l_c(t) \simeq 2\Gamma(0)t^{1/2}$. In this case the limits $a \to 0$ and $t \to \infty$ commute and the disorder is called weak. The situation is very different for the strong disorder case $0 < \alpha < 1$ because the diffusion coefficient is zero and a generalized frequency coefficient $\Gamma(s)$ must be considered. Now taking the limit $a \to 0$ before $t \to \infty$, one obtains the static limit in which particles are at rest. The limit observed in physical systems, $t \to \infty$ before $a \to 0$, leads to the frequency-dependent coefficient $\Gamma(s) \sim s^\mu$ with $\mu = \alpha/(2-\alpha)$ [24].

Returning to the calculation of the interface width $\sigma^2(l,t)$, we substitute (7) in (6) and use the scaling $\Gamma(\lambda s) \sim \lambda^\mu \Gamma(s)$, obtaining

$$\sigma^2(l,s) \simeq \frac{\theta s^{-(5+\mu)/2}}{4ls^{(1-\mu)/2}}\left(\int_0^{ls^{(1-\mu)/2}} \psi(x,x)dx - \frac{1}{ls^{(1-\mu)/2}}\int_0^{ls^{(1-\mu)/2}} dx \int_0^{ls^{(1-\mu)/2}} dy\psi(x,y)\right), \quad (9)$$

where the function ψ is given by

$$\psi(x,y) = \int_{-\infty}^{\infty} dx_1 \int_0^1 \frac{ds_1}{(1-s_1)^2 s_1^2 \Gamma(1-s_1)\Gamma(s_1)} \times \exp\{-[(1-s_1)/\Gamma(1-s_1)]^{1/2}|x-x_1| - [s_1/\Gamma(s_1)]^{1/2}|y-x_1|\}.$$

Now the saturation time can be defined by the condition $ls^{(1-\mu)/2} \sim s^{-1}$, so we find that $t_s(l) \sim l^{(2-\alpha)/(1-\alpha)}$. At times $t \ll t_s$ but large enough to prevent transients ($s^{-1} \gg 0$, $ls^{(1-\mu)/2} \to \infty$), we have $\sigma^2(s) \sim s^{-(5-2\alpha)/(2-\alpha)}$. In terms of time $\sigma^2(t) \sim t^{(3-\alpha)/(2-\alpha)}$, which implies $\beta = \frac{3-\alpha}{2(2-\alpha)}$. As usual, the roughness exponent is obtained in the saturation regime when the horizontal correlation length $l_c(t) \sim t^{(1-\alpha)/(2-\alpha)}$ reaches the size l. Hence, for $t \gg t_s$, we have $\sigma^2(l) \sim t_s(l)^{2\beta} \sim l^{(3-\alpha)/(1-\alpha)}$ and consequently $\chi = \frac{(3-\alpha)}{2(1-\alpha)}$ is the roughness exponent in the strong disorder case. These exponents are our main results; a disorder with finite correlation length, given by the lattice spacing a, leads to a growth with characteristic exponents depending on the disorder.

Although the above exponents have been calculated exactly, we would like to show how they may also be obtained by a scaling argument. Disorder in the coefficient $D(x)$ of (1) allows one to define an horizontal characteristic length $l_c(t) \sim t^{(1-\alpha)/(2-\alpha)}$ according to the fact that $\Gamma(s) \sim s^{\mu}$ is the generalized diffusion coefficient and $\mu = 2/(2-\alpha)$ in our model. On the other hand, the height dispersion in a set of channels with random quenched velocity $\eta(x)$ has already been calculated in Ref. [25] and is given by $\langle h^2\rangle^{1/2}(t) \sim \langle \eta(x)^2\rangle^{1/2} t$. Since $\eta(x)$ is a Gaussian noise and the number of channels involved until time t scales as $t^{\frac{1-\alpha}{2(2-\alpha)}}$, we have $\langle \eta(x)^2\rangle^{1/2} \sim t^{\frac{\alpha-1}{2(2-\alpha)}}$ and consequently the interface width scales as $\sigma(t) \sim \langle h^2\rangle^{1/2}(t) \sim t^{\frac{(3-\alpha)}{2(2-\alpha)}}$, as obtained in the previous analysis by more rigorous methods.

III. NUMERICAL RESULTS

We have simulated the discretized version (2) of (1) for $F = 0$ and calculated the exponents β and χ numerically. The numerical integration of (2) was carried out using a lattice spacing $a = 1$ and large system size $L = 1000, 5000, 10\,000$. In order to produce simulations for long times we used time steps $\Delta t = 0.01$ and $\Delta t = 0.001$. The appropriate Δt depends on the actual values of $D_{\max}$ and the intensity of noise θ. We checked that the numerical results do not change by making runs for smaller Δt. The temporal exponent β was always obtained with very good accuracy and short computer time averaging over 15 realizations of disorder. In Fig. 1 the results from simulations together with the theoretical value for several degrees of disorder are presented.

The roughness exponent χ had to be treated with care because in the quenched columnar case the growth is very dependent on the particular realization of disorder. In a numerical simulation, the measure of the exponent χ is usually a rather time consuming task since an interface with size L reaches the saturation regime at times that increase with L as $L^{\chi/\beta}$. There is a long wait for the saturation of the interface. In order to shorten the computer time, an alternative method is often used that is expected to lead to the same final result: an average over all pieces of length $l \ll L$ is taken in the same realization and then, the average over realizations is performed. By following this method we obtained a roughness exponent $\chi = 0.94 \pm 0.03$ for constant diffusivity $\chi = 0.96 \pm 0.05$ for $\alpha = -1000$ (weak disorder) and $\chi = 0.74 \pm 0.08$

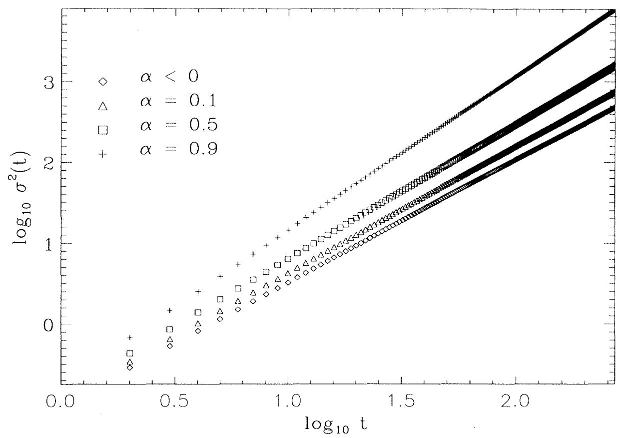

FIG. 1. Square width versus time in a log-log plot for four differents degrees of disorder. The slopes of the curves are 1.51 ± 0.01 for $\alpha < 0$ and 1.57 ± 0.03, 1.678 ± 0.002, and 1.906 ± 0.002 for strong disorder with α equal to 0.1, 0.5, and 0.9, respectively. The analytical value for 2β is 3/2 for weak disorder and 1.53, 1.67, 1.91 for strong disorder with parameter α equal to 0.1, 0.5, and 0.9, respectively.

for $\alpha = 0.1$ (strong disorder). This technique has been widely used in the literature [1,6,9,10] to determine the roughness exponent, but we will show that it is inappropriate in our case. In our simulations we observed that the columnar problem posed in (1) presents strong dispersion of the saturation time for pieces of interface with the same length l. This dispersion exists even when one maintains both fields $D(x)$ and $\eta(x)$ inside a piece of length l and changes the fields in the rest of the substrate, reflecting an extreme dependence on boundary conditions. In a problem of growing interfaces with disorder that changes in time [26], the saturation time of a piece with size l is a well defined quantity (with small dispersion between realizations) due to the self-averaging that is induced by temporal fluctuations. However, in our model the saturation time of a little piece of interface is not a self-averaging quantity and the method described below, which is useful in problems driven by dynamical noise, gives incorrect results in our case. So, to determine the correct value of the roughness exponent we simulated Eq. (2) for substrates with different sizes ($L = 100$, 250, 350, 500, and 700) and obtained the interface width in the saturation regime. We averaged over 15 realizations of disorder with a high cost of computing time (see Fig. 2). The exponent $\chi = 1.43 \pm 0.09$ was obtained in the constant diffusivity case [$D(x) = D$] and also for $\alpha < 0$, which corresponds to weak disorder. For strong disorder $\alpha = 0.1$, the roughness measured was $\chi = 1.58 \pm 0.06$ and $\chi = 1.76 \pm 0.09$ for stronger disorder $\alpha = 0.2$. In all cases, very good agreement with the predicted value $\chi = \frac{3-\alpha}{2(1-\alpha)}$ was found. Because computing time greatly increases as disorder becomes stronger, we were not able to explore greater values of α.

Finally, we have studied a case in which disorders are correlated i.e., $\langle D(x)\eta(x)\rangle \neq 0$. From an intuitive point of view, it would seem that strong correlations between random pinning force and diffusivity leads to changes in

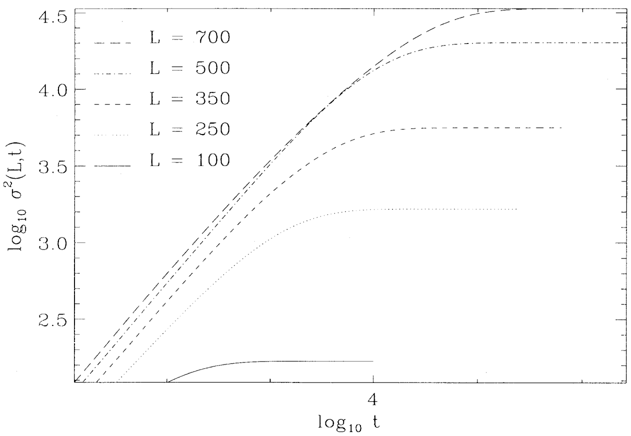

FIG. 2. Saturation regime (in the weak disorder case) of the interface for five systems with different sizes. The value of the width in the saturation regime is taken to fit the scaling law $\sigma(L) \sim L^{\chi}$. Similar plots are obtained for the saturation of the width in the cases of strong disorder.

the roughness exponents. However, from analytical considerations one can see that corrections due to correlations do not appear in the exponents of the leading term, but they change the coefficients of (7). Of course, this happens whenever the resummed expansion (7) is convergent. In our simulations we have introduced a very strong correlation as follows. Let us define the median D_m of the distribution function of D as $\int_0^{D_m} P(D)dD = 1/2$. Given a position x, we take $D(x)$ to be distributed according to $P(D) \sim D^{-\alpha}$. Then, the random field $\eta(x)$ is chosen $\eta(x) > 0$ if $D(x) > D_m$ and $\eta(x) < 0$ if $D(x) < D_m$. In this way the Gaussian property of $\eta(x)$ is maintained and a strong constraint is included. In agreement with analytical considerations, the exponents obtained in our simulations do not change significantly in any case. Note that the correlation was strong but short ranged (disorders are correlated at position x).

IV. CONCLUSION

In summary, we have studied the effect of quenched columnar disorder in growing interfaces. We have shown how disorder in the diffusivity changes the characteristic exponents of growing. In the weak disorder case or when the diffusivity is constant, the trivial exponents $\beta = 3/4$ and $\chi = 3/2$ are found. However, in the strong disorder case, these exponents depend on a parameter α that characterizes the intensity of disorder. In the former case, by using standard methods in transport theory we also obtained exactly the exponents $\beta = \frac{3-\alpha}{2(2-\alpha)}$ and $\chi = \frac{3-\alpha}{2(1-\alpha)}$ in agreement with simulations. The exponents are unchanged when a short-range correlation is included between the columnar pinning force and the random diffusivity. Our results indicate an interesting case of growth in which a quenched disorder with short correlation length leads to anomalous exponents β and χ. A different model of growth with columnar disorder was recently considered in Ref. [19]. The authors numerically studied an approach to explain the experiments on kinetic roughening by considering a stochastic differential equation with multiplicative noise. The effect of multiplicative noise can be thought of as similar to a random diffusivity. However, both models are rather differents due to the high nonlinearity of the equation studied in Ref. [19] (the term $[1 + (\nabla h)^2]^{1/2}$ was included in its full form), contrary to the linear character of our model.

The model discussed in this paper is used in fluid flow through some random media when correlations over a large distance appear. Also, columnar disorder is very important in the roughening of directed polymers in random media where its effect at zero temperature is to break down the universality property expected to exist and a strong sample to sample dispersion of the globally optimal path is found [20].

ACKNOWLEDGMENT

Financial support from Dirección General de Investigación Científica y Técnica (Spain), Project No. PB93-0054-C02-02, is acknowledged.

[1] T. Halpin-Healy and Y.-C. Zhang, Phys. Rep. **254**, 215 (1995).
[2] T. Halpin-Healy, Phys. Rev. Lett. **62**, 442 (1989); Phys. Rev. A **42**, 711 (1990).
[3] Y. Imry, J. Stat. Phys. **6**, 849 (1984).
[4] M. Kardar, G. Parisi, and Y.-C. Zhang, Phys. Rev. Lett. **56**, 889 (1986).
[5] S.F. Edwards and D.R. Wilkinson, Proc. R. Soc. London, Ser. A **381**, 17 (1982).
[6] J. Krug and H. Spohn, in *Solids Far From Equilibrium: Growth, Morphology and Defects*, edited by C. Godréche (Cambridge University Press, Cambridge, 1991).
[7] J. Koplik and H. Levine, Phys. Rev. B **32**, 280 (1985).
[8] O. Narayan and D.S. Fisher, Phys. Rev. B **48**, 7030 (1993).
[9] D.A. Kessler, H. Levine, and Y. Tu, Phys. Rev. A **43**, R4551 (1991).
[10] L.A.N. Amaral, A.-L. Barabási, and H.E. Stanley, Phys. Rev. Lett. **73**, 62 (1994).
[11] J.M. López, M.A. Rodríguez, A. Hernández-Machado, and A. Díaz-Guilera, Europhys. Lett. **29**, 197 (1995).
[12] T. Nattermann, S. Stepanow, L.-H. Tang, and H. Leschhorn, J. Phys. (France) II **2**, 1483 (1992).
[13] L.-H. Tang, M. Kardar, and D. Dhar, Phys. Rev. Lett. **74**, 920 (1995).
[14] E. Medina, T. Hwa, M. Kardar, and Y.-C. Zhang, Phys. Rev. A **39**, 3053 (1989).
[15] M.H. Jensen and I. Procaccia, J. Phys. (France) II **1**, 1139 (1991).
[16] L.-H. Tang and H. Leschhorn, Phys. Rev. A **45**, R8305 (1992); S.V. Buldyrev, A.-L. Barabási, F. Caserta, S. Havlin, H.E. Stanley, and T. Vicsek, *ibid.* **45**, R8313

(1992); Kim Sneppen, Phys. Rev. Lett. **69**, 3539 (1992).
[17] P. Le Doussal, J. Stat. Phys. **69**, 917 (1992).
[18] G. Matheron and G. de Marsilly, Water Resour. Res. **16**, 901 (1980).
[19] T. Vicsek, E. Somofai, and M. Vicsek, J. Phys. A **25**, L763 (1992).
[20] L.-H. Tang and I.F. Lyuksyutov, Phys. Rev. Lett. **71**, 2745 (1993); I. Arsenin, T. Halpin-Healy, and J. Krug, Phys. Rev. E **49**, R3561 (1994).
[21] S. Alexander, J. Bernasconi, W.R. Schneider, and R. Orbach, Rev. Mod. Phys. **53**, 175 (1981).
[22] J.P. Bouchaud and A. Georges, Phys. Rep. **195**, 127 (1990).
[23] M.A. Rodríguez, E. Hernández-García, L. Pesquera, and M. San Miguel, Phys. Rev. B **40**, 4212 (1989); E. Hernández-García, M.A. Rodríguez, L. Pesquera, and M. San Miguel, *ibid.* **42**, 10 653 (1990).
[24] J.M. López, M.A. Rodríguez, and L. Pesquera, Phys. Rev. E **51**, R1637 (1995).
[25] J.-P. Bouchaud, A. Georges, J. Koplik, A. Provata, and S. Redner, Phys. Rev. Lett. **64**, 2503 (1990).
[26] E.g., for thermal fluctuations $\eta(x,t)$ or uniform disorder $\eta(x,h(x,t))$. In the former case η depends on $h(x,t)$ and the interface dynamically explores the disorder.